Dispersion Characteristics of Moderately Thick Microstrip Lines by the Spectral Domain Method

R. T. Kollipara and V. K. Tripathi

Abstract— **The influence of the metallization thickness on the propagation characteristics of the microstrip lines is modeled by utilizing the computationally efficient spectral domain method with an approximate Green's function. The Green's function is based on the current and charge distribution at the top and bottom surfaces of the microstrips. The calculated effective dielectric constants and impedances are plotted for single and coupled microstrip lines as a function of frequency. It is seen that the effective dielectric constants obtained from resonance measurements are in good agreement with the calculated values for moderately thick ($t/W \lesssim 0.15$) single and coupled lines.**

I. Introduction

The spectral domain technique has proven to be accurate and efficient for the computation of the propagation characteristics of various planar transmission line structures of zero thickness metallization. Recently, the quasi-TEM spectral domain analysis of thick planar transmission lines with a modified Green's function was shown to yield accurate results [1]–[3]. The method has also been applied to microstrip structures having trapezoidal conductor cross sections [3]. Recently, an eigen function weighted boundary integral equation method [4] has been used to analyze the dispersion characteristics of thick transmission lines. Even though the three-dimensional full wave methods including the integral equation method lead to more accurate results, the results based on a two-level charge or current distribution have been shown to be quite accurate for small to moderately thick strips [1]–[3]. In this letter, the spectral domain technique with a modified Green's function is used to analyze the full-wave characteristics of single and coupled microstrip lines with thick conductors. The computed values of the effective dielectric constants as a function of frequency are shown to be in good agreement with the values obtained from the resonance measurements.

II. Analysis

A symmetric coupled bounded microstrip lines, with metallization thickness t, are shown in Fig. 1. The spectral domain technique is used to calculate the surface current density distribution on both the top and the bottom surfaces of each conductor by using the spectral Green's function for multilayered structures and the Galerkin's procedure [5]. If the currents are assumed to be at the bottom and the top surfaces of the conductors, their values are related to the corresponding electric fields at the two levels by

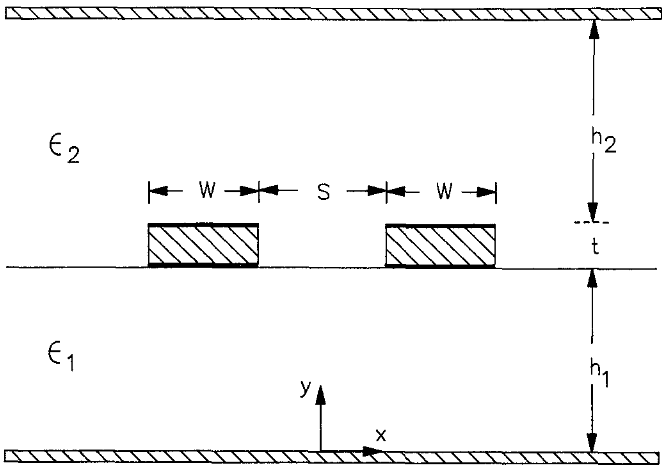

Fig. 1. Covered symmetric coupled microstrip transmission line structure.

$$\begin{bmatrix} \tilde{J}_z(\alpha, h_1) \\ \tilde{J}_x(\alpha, h_1) \\ \tilde{J}_z(\alpha, h_1+t) \\ \tilde{J}_x(\alpha, h_1+t) \end{bmatrix} = \begin{bmatrix} \tilde{Y}_{11}^{zz} & \tilde{Y}_{11}^{zx} & \tilde{Y}_{12}^{zz} & \tilde{Y}_{12}^{zx} \\ \tilde{Y}_{11}^{xz} & \tilde{Y}_{11}^{xx} & \tilde{Y}_{12}^{xz} & \tilde{Y}_{12}^{xx} \\ \tilde{Y}_{21}^{zz} & \tilde{Y}_{21}^{zx} & \tilde{Y}_{22}^{zz} & \tilde{Y}_{22}^{zx} \\ \tilde{Y}_{21}^{xz} & \tilde{Y}_{21}^{xx} & \tilde{Y}_{22}^{xz} & \tilde{Y}_{22}^{xx} \end{bmatrix} + \begin{bmatrix} \tilde{E}_z(\alpha, h_1) \\ \tilde{E}_x(\alpha, h_1) \\ \tilde{E}_z(\alpha, h_1+t) \\ \tilde{E}_x(\alpha, h_1+t) \end{bmatrix}. \quad (1)$$

The impedance matrix relating tangential electric fields to currents at the boundary are given by a similar expression. These boundary dyadic Green's function elements are readily found by using the spectral domain formulation for the multilayered structures such as the one given in [5]. These elements of the $[Y]$ matrix are found to be

$$\tilde{Y}_{11,22}^{zz} = \left[\beta^2 Y_{e1,2} + \alpha^2 Y_{h1,2}\right] / (\alpha^2 + \beta^2) \quad (2a)$$

$$\tilde{Y}_{11,22}^{xx} = \left[\alpha^2 Y_{e1,2} + \beta^2 Y_{h1,2}\right] / (\alpha^2 + \beta^2) \quad (2b)$$

$$\tilde{Y}_{11,22}^{zx} = \tilde{Y}_{11,22}^{xz} = \alpha\beta\left[Y_{e1,2} - Y_{h1,2}\right] / (\alpha^2 + \beta^2) \quad (2c)$$

$$\tilde{Y}_{12}^{zz} = \tilde{Y}_{21}^{zz} = -\left[\beta^2 Y_{e3} + \alpha^2 Y_{h3}\right] / (\alpha^2 + \beta^2) \quad (2d)$$

$$\tilde{Y}_{12}^{xx} = \tilde{Y}_{21}^{xx} = -\left[\alpha^2 Y_{e3} + \beta^2 Y_{h3}\right] / (\alpha^2 + \beta^2) \quad (2e)$$

$$\tilde{Y}_{12}^{zx} = \tilde{Y}_{12}^{xz} = \tilde{Y}_{21}^{zx} = \tilde{Y}_{21}^{xz} \quad (2f)$$

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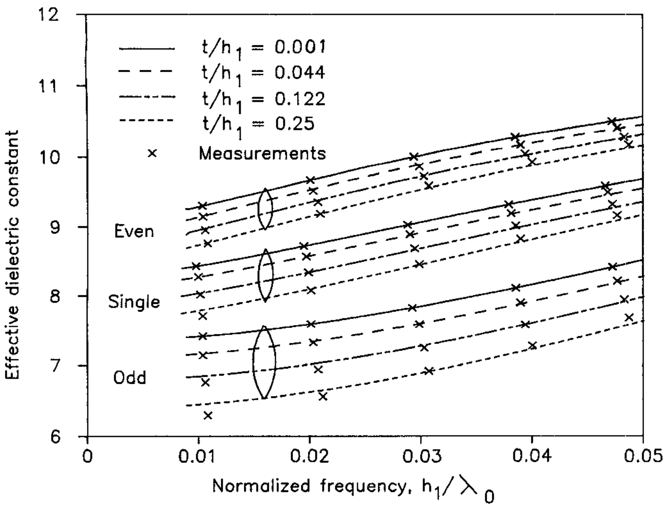

Fig. 2. Effective dielectric constants of single and coupled open microstrip lines. ($\varepsilon_r = 12.5$, $W/h_1 = 1$ and $S/W = 1$ for coupled lines).

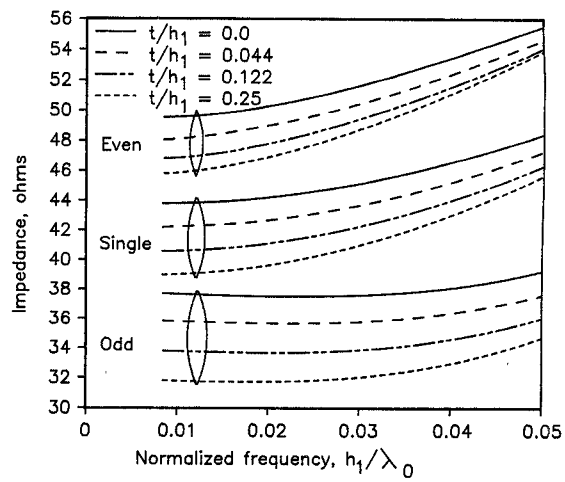

Fig. 3. Characteristic impedances of single and coupled open microstrip lines. ($\varepsilon_r = 12.5$, $W/h_1 = 1$ and $S/W = 1$ for coupled lines).

$$= -\alpha\beta[Y_{e3} - Y_{h3}]/(\alpha^2 + \beta^2), \quad (2g)$$

with

$$Y_{e1} = j\omega\varepsilon_1 \coth(\gamma_1 h_1)/\gamma_1 + j\omega\varepsilon_t \coth(\gamma_t h_t)/\gamma_t \quad (3a)$$
$$Y_{e2} = j\omega\varepsilon_2 \coth(\gamma_2 h_2)/\gamma_2 + j\omega\varepsilon_t \coth(\gamma_t h_t)/\gamma_t \quad (3b)$$
$$Y_{e3} = j\omega\varepsilon_t/[\gamma_t \sinh(\gamma_t h_t)] \quad (3c)$$
$$Y_{h1} = \gamma_1 \coth(\gamma_1 h_1)/(j\omega\mu_0) + \gamma_t \coth(\gamma_t h_t)/(j\omega\mu_0) \quad (3d)$$
$$Y_{h2} = \gamma_2 \coth(\gamma_2 h_2)/(j\omega\mu_0) + \gamma_t \coth(\gamma_t h_t)/(j\omega\mu_0) \quad (3e)$$
$$Y_{h3} = \gamma_t/[j\omega\mu_0 \sinh(\gamma_t h_t)] \quad (3f)$$
$$\gamma_{1,2,t}^2 = \alpha^2 + \beta^2 - \omega^2\mu_0\varepsilon_{1,2,t}. \quad (3g)$$

The expressions for the impedance matrix elements can be found directly by using the same procedure or by inverting the previous matrix, i.e., $[Z] = [Y]^{-1}$. Expanding the same surface currents in suitable basis functions and using the standard Galerkin procedure leads to the propagation constants β and effective dielectric constants. We have used the Chebyshev polynomials of first and second kind with suitable edge terms to represent these surface currents [5]. Note that the total current for each strip is the sum of corresponding top and bottom surface currents. The characteristic impedance is calculated as the ratio of the power transmitted in the z-direction to the square of the current flowing in the z-direction [5].

III. Results and Conclusion

The calculated effective dielectric constants for single and coupled microstrip lines as a function of normalized frequency are plotted in Fig. 2 for various values of t/h_1. The effective dielectric constants obtained from the standard resonance measurements are also shown in the figure. As seen from the figure, the calculated values are in good agreement with the measured values for $t/W \lesssim 0.15$ and start deviating from the measured values for higher metallization thicknesses. Note that the currents on the side walls are neglected in this two level approximate formulation which limits the validity of the model to $t/W \lesssim 0.15$ for single lines and corresponding limitations ($t/W \lesssim 0.15$ and $t/S \lesssim 0.15$) for coupled lines. This is consistent with the quasi-static results where the two level model has been shown to be in good agreement with the multilevel model for similar thick strips [2]. The impedances for a typical case of single and coupled microstrip lines as a function of frequency for various values of strip thicknesses are shown in Fig. 3.

In conclusion, a computationally efficient spectral-domain method with an approximate, two level Green's function is used to analyze the propagation characteristics of moderately thick single and coupled microstrip transmission lines. For microstrips used in (M)MIC's with $t/W \lesssim 0.15$, the technique presented here is quite accurate, efficient and compatible with the circuit analysis based CAD tools. The same procedure can also be used for other planar transmission line structures with moderate conductor thicknesses.

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